Unpublished Appendix

"Indirect Network Effects and the Product Cycle: Video Games in the U.S., 1994-2002"

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Abstract

This unpublished appendix is organized as follows: Section A shows how to derive the hardware adoption model (1) in the main text. Sections B derives the nested logit demand model in (2), and Section C calculates the elasticities of hardware demand with respect to price and installed base, respectively. Section D discusses the derivation of the software entry model, and Section E contains additional tables and figures.

A Derivation of Hardware Utility (1)

This appendix presents a hardware adoption model that led us to formulating the utility function (1) introduced in Section 3.1. Following the theoretical work on indirect network effects,[1] we begin with consumer preferences over hardware and software. There are two types of goods in the economy: video game systems and the outside alternative. Our study used the television household as the purchasing entity. We also assumed that a household has a unit demand for a video game console. A representative consumer is assumed to maximize the following quasi-linear indirect utility function, U_j, by buying a console type j (we omit the time subscript here):

$$U_j = z_j + q_0 \tag{A1}$$

$$\equiv F\left[\left(\sum_{s=1}^{N_j} d_{sj}^{1/\tau}\right)^{\tau}\right] + q_0, \tag{A2}$$

where $\tau > 1$, q_0 is the consumption amount of the outside good, z is the sub-utility accrued from the variety in game titles, d_{sj} is the consumption amount of game title s compatible with the system j, and N_j is the variety of game titles available for console j. The CES utility function is often used in modelling variety in z. Two important assumptions are embedded in this utility function to impose some restrictions on software demand. The first assumption is that consumption of the outside good is included in an additively separable way. This assumption guarantees that the software demand, d_{sj}, is independent of the income effect. This

[1] The work includes Church and Gandal (1992; 1993) and Chou and Shy (1990).

is perhaps a reasonable description in the U.S. video game market, because titles are inexpensive: The sales-weighted average of software price is merely \$27 per title. The second assumption is that, following Park (2002), we use an increasing and concave function, F, to transform the CES. As discussed in Park (2002), some restriction is necessary on F for the optimal software demand to be a function of the software variety, N_j. If the software demand is independent of N_j, an individual software price, and hence its profit, do not change with the amount of entry. The free-entry condition in this case determines the optimal variety as being either zero or infinity. This implication obviously contradicts the data. In the remainder of this appendix, we restrict the function as $F(A) = A^{1/(2\tau)}$, and proceed with the discussion. A similar assumption is used in Nair et al (2003).

If a household decides not to purchase a game system, they only consume the outside good, q_0. The representative household faces a budget constraint:

$$\sum_{s=1}^{N_j} \rho_s^j d_{sj} + q_0 + p_j = y, \tag{A3}$$

where ρ_s^j is the price of software variety s on console j, p_j is the price of console j, and y is a representative consumer's expenditure on a game system and the outside good. The timing of the game is as follows: Given a hardware price, each consumer decides whether to buy a game system, and if they buy, which console to buy. They choose the action that provides them with a higher expected utility. Since the console itself has no entertainment value, those households who buy a console purchase game titles, given the available number of titles compatible with the game system. This software variety is in turn determined by the entry of software firms, as we describe shortly in this appendix. We solve this consumer decision problem by backward induction.

A consumer who purchases console j chooses d_{sj} and q_0 to maximize U_j under the budget constraint. The software demand is derived as

$$d_{sj}^* = (2\tau Q_j)^{2\tau/(1-2\tau)} \left(\frac{Q_j}{\rho_s^j}\right)^{\tau/(\tau-1)}, \tag{A4}$$

where $Q_j = \left(\sum_{s=1}^{N_j} \left(\rho_s^j\right)^{1/(1-\tau)}\right)^{1-\tau}$. Following the treatment in the literature, we focus on the case in which the price of software supplied to each console is the same, i.e. $\rho_s^j = \rho^j$. The symmetric software demand d_j^* is

$$d_j^* = \left(2\tau\rho^j\right)^{2\tau/(1-2\tau)} (N_j)^{\tau/(1-2\tau)}. \tag{A5}$$

The symmetric demand is free of the income effect (i.e., $y - p_j$), and is a function of software variety, as we expect from the assumptions on (A1). A consumer decides the amount of d_j^*, anticipating the software suppliers' response in the price, ρ^j, and the number of titles available in the market, N_j. Considering their software demand in the later periods, the representative household obtains the following indirect utility function, U_j^*, if they purchase a console type j:

$$U_j^* = y - p_j + h(N_j), \tag{A6}$$

where $h(N_j) = \left[(2\tau)^{1/(1-2\tau)} - (2\tau)^{2\tau/(1-2\tau)}\right] \left(\rho^j\right)^{1/(1-2\tau)} (N_j)^{(1-\tau)/(1-2\tau)}$. The software price, ρ^j, is determined by a software provider's profit maximization problem. Our estimation model is based on (A6).

B Derivation of Demand Model (2)

The nested logit introduced in Section 3.1 gives a closed form choice probability. The probability of purchase at time t is:

$$s_{B_t=1} = \frac{D_{B_t=1}^{(1-\sigma)}}{\left[\sum_{B_t=0,1} D_{B_t}^{(1-\sigma)}\right]}.$$

where B_t takes 1 when console is purchased, and 0 otherwise, and $D_{B_t=1} = \sum_{j\in\vartheta_{B_t=1}} e^{\delta_j/(1-\sigma)}$. The probability of choosing a brand j given the decision to purchase $(B_t = 1)$ is

$$s_{j/B_t=1} = \frac{e^{\delta_j/(1-\sigma)}}{D_{B_t=1}}.$$

The market share of product j is $s_j = s_{j/B_t=1} \cdot s_{B_t=1}$, which is

$$s_j = \frac{e^{\delta_j/(1-\sigma)}}{D_{B_t=1}^{\sigma}\left[\sum_{B_t=0,1} D_{B_t}^{(1-\sigma)}\right]}.$$

The market share for the outside good $(j = 0)$ is (Note that $D_{B_t=0} = e^{\delta_0/(1-\sigma)}$),

$$s_0 = \frac{e^{\delta_0}}{\sum_{B_t=0,1} D_{B_t}^{(1-\sigma)}},$$

Equation (2) on p.9 can be derived as follows. First take logs for s_j and s_0, and subtract one from the other:

$$\ln(s_j) - \ln(s_0) = \frac{\delta_j}{1-\sigma} - \delta_0 - \sigma \ln\left(D_{B_t=1}\right),$$

where δ_0 is the mean utility of consuming the outside good. Replace $\ln\left(D_{B_t=1}\right)$ by using the equation of $s_{j/B_t=1}$ above to derive the equation (2).

C Derivation of Own-Price and Installed-Base Derivatives

It is convenient to rewrite the choice probabilities of $s_{B_t=1}$, $s_{j/B_t=1}$, and s_j before computing the derivatives. The probability of purchase, $B_t = 1$, is (dropping the time subscript):

$$s_{B_t=1} = \frac{D_{B_t=1}^{(1-\sigma)}}{\left[\sum_{B_t=0,1} D_{B_t}^{(1-\sigma)}\right]}.$$

where $D_{B_t=1} = \sum_{j\in\vartheta_{B_t=1}} e^{\delta_j/(1-\sigma)}$. The probability of choosing a brand j given the decision to purchase is

$$s_{j/B_t=1} = \frac{e^{\delta_j/(1-\sigma)}}{D_{B_t=1}}.$$

The market share of product j is $s_{jt} = s_{jt/B_t=1} \cdot s_{B_t=1}$, which is

$$s_j = \frac{e^{\delta_j/(1-\sigma)}}{D^{\sigma}_{B_t=1}\left[\sum_{B_t=0,1} D^{(1-\sigma)}_{B_t}\right]}.$$

The market share for the outside good ($j = 0$) is,

$$s_0 = \frac{e^{\delta_0}}{\sum_{B_t=0,1} D^{(1-\sigma)}_{B_t}},$$

Now, the derivative for product $i \in B_t = 1$ with respect to own price is:

$$\begin{aligned}\frac{\partial s_i}{\partial p_i} &= \frac{\partial \delta_i}{\partial p_i} \cdot \left(\frac{1}{1-\sigma} s_i - s_i \cdot \frac{\sigma \cdot e^{\delta_j/(1-\sigma)}}{(1-\sigma) \cdot D_{B_t=1}} - (s_i)^2\right) \\ &= \frac{\beta_p}{1-\sigma} s_i \cdot \left(1 - \sigma \cdot s_{i/B_t=1} - (1-\sigma)\, s_i\right),\end{aligned}$$

where δ_i is defined on p.8. Likewise the derivative of s_j with respect to the installed base variable, IB_j, becomes:

$$\begin{aligned}\frac{\partial s_i}{\partial IB_i} &= \frac{\partial \delta_i}{\partial IB_i} \cdot \left(\frac{1}{1-\sigma} s_i - s_i \cdot \frac{\sigma \cdot e^{\delta_j/(1-\sigma)}}{(1-\sigma) \cdot D_g} - (s_i)^2\right) \\ &= \frac{\omega h'(N_g)}{1-\sigma} s_i \cdot \left(1 - \sigma \cdot s_{i/g} - (1-\sigma)\, s_i\right).\end{aligned}$$

D Derivation of Software Entry Model (3)

This section again uses the theoretical model often used in the literature of indirect network effects (see footnote 1 in this appendix). To simplify the estimation model, we assume a single-product software firm provides its game title to a console $j \in J_t$, where J_t is the number of consoles available at t). [2] Those consumers who purchase game titles already own a console. The market size for the software is thus the size of the installed base, IB_{gt}. We use the index g to account for the backward compatibility of the PS2, already mentioned in the previous section. Each consumer in the installed base of console j has a demand for software s. As derived in Section 1, this demand at time t, d^*_{sjt}, decreases with the software price, ρ^j_{st}, and increases with the aggregate software price index for console j, Q_{jt}. Facing the software demand, a representative firm s maximizes profit, π^j_{st}, at time t:

$$\pi^j_{st} = IB_{gt} \cdot d^*_{sjt}\left(\rho^j_{st}, Q_{jt}\right) \cdot \left(\rho^j_{st} - mc_j\right) - F_j,$$

where mc_j is the marginal cost of providing a game title compatible with console j. This term includes the cost for production, delivery, and packaging, and a royalty fee paid to the console provider j. Let F_j be the fixed cost of introducing a game title. The marginal and fixed costs are assumed constant over time. We, however, allow for the possibility that there are console-specific elements in the fixed and marginal costs.

[2] We believe this to be an innocuous assumption. There is a large fixed cost involved in developing a game title, and no significant economies of scope are present in the production of multiple titles.

For example, developing games for the PS is on average more expensive than developing games for the NES, because of the greater complexity allowed by the hardware.[3]

Software firm s chooses ρ_{st}^{j} to maximize its profit. We assume Bertrand competition where a software supplier takes the price of other software titles as given. Since more than 1000 titles were available in any given year in our sample (see Table 1), the degree of dependence of ρ_{st}^{j} on Q_{jt} would have been very small. If we assume this dependence to be zero (we discuss the case otherwise below), the symmetric equilibrium software price is $\rho_{t}^{j} = \tau mc_j$. Thus the equilibrium profit of a representative software provider is

$$\pi_t^j = \Phi_j^{(1/\gamma)} \cdot IB_{gt} \cdot N_{jt}^{-(1/\gamma)} - F_j,$$

where $\gamma = \frac{2\tau - 1}{\tau}$ and $\Phi_j = \left[(\tau - 1)\, mc_j \left(2\tau^2 mc_j\right)^{2\tau/(1-2\tau)}\right]$. A free-entry condition requires that the number of software firms is determined by the equilibrium in which a representative firm makes zero profit. Therefore the equilibrium number of firms, which is also the degree of available variety in game titles, is

$$N_{jt} = A_j \cdot (IB_{gt})^{\gamma}, \tag{A7}$$

where $A_j = \Phi_j (F_j)^{-\gamma}$. We thus use the following empirical model:

$$\ln(N_{jt}) = \alpha_j + \gamma \ln(IB_{gt}) + \eta_{jt}, \tag{A8}$$

where η_{jt} is a mean-zero error. Although we do not have data on fixed and marginal costs of production for game titles by console, we use a console fixed effect to take care of $\alpha_j \equiv \ln(A_j)$. We assume that the installed base for console j, IB_{gt}, is a cumulative sum of console sales up to the time $t-1$, and we consider the case of depreciation in the estimation.

To derive (A8), we assume that the derivative of Q_{jt} with respect to ρ_{st}^{j} is zero. When we consider this derivative explicitly, the symmetric equilibrium software price becomes $\rho_t^j = 2\tau mc_j$, and the equilibrium number of firms is still the same form as (A7); the only difference is $\Phi_j = \left[(2\tau - 1)\, mc_j \left(4\tau^2 mc_j\right)^{2\tau/(1-2\tau)}\right]^{\gamma}$. The estimation method is discussed in Section 4.2, and the results are in Section 5.

E Tables and Figures

Table A1: Appendix to Table 2 (Full estimation results)

Figure A1: Rivalry in the Other Three U.S. Video Game Systems (Appendix to Figure 1)

Figure A2: Network Effects in the Hardware Market (Appendix to Figure 2)

Figure A3: Indirect Network Effect in the Software Market (Appendix to Figure 3)

Figure A4: Indirect Network Effects in the Software Market for Major Consoles (without use of hardware age)

Figure A5: Market Share and Price for Major Consoles Introduced after 1994

[3] According to Coughlan (2001), the average cost of developing a title for an 8-bit console like the NES was \$80,000. The average cost for a 32-bit console like the PS was \$1.5 million.

TABLE A1
(Appendix to TABLE 2)

Hardware Estimation Results

	(H1)		(H2)		(H3)		(5% depreciation)		(J)	
	est	w.std	est	w.std	est	w.std	est	w.std	est	w.std
one	-16.03	2.35	-16.40	1.87	-16.37	0.96	-16.40	1.87	-16.40	1.56
h_price	-0.70	0.59	-0.71	0.25	-0.50	0.15	-0.71	0.25	-0.71	0.22
h_soft_N	0.41	0.51	0.41	0.17	0.43	0.09	0.41	0.17	0.41	0.14
ln(sj|B=1)	0.35	0.12	0.35	0.09	0.60	0.03	0.35	0.09	0.35	0.06
1994	8.32	1.31	8.30	1.26	4.65	0.67	8.30	1.26	8.30	0.98
1995	8.08	1.07	8.06	1.20	4.75	0.61	8.06	1.20	8.06	0.90
1996	8.03	0.87	8.02	1.15	4.89	0.59	8.02	1.15	8.02	0.84
1997	6.64	0.73	6.63	0.94	4.32	0.52	6.63	0.94	6.63	0.71
1998	4.57	0.53	4.56	0.61	2.73	0.36	4.56	0.61	4.56	0.48
1999	3.09	0.35	3.08	0.43	1.73	0.28	3.08	0.43	3.08	0.35
2000	1.47	0.27	1.46	0.30	0.70	0.21	1.46	0.30	1.46	0.25
94*Genesis	-4.60	0.53	-4.60	0.69	-3.19	0.45	-4.60	0.69	-4.60	0.52
94*SNES	-2.43	0.78	-2.43	0.52	-1.72	0.42	-2.43	0.52	-2.43	0.45
95*PS	-2.89	1.82	-2.89	1.56	0.20	0.98	-2.89	1.56	-2.89	1.42
95*Genesis	-5.40	0.75	-5.39	0.91	-4.26	0.49	-5.39	0.91	-5.39	0.65
95*SNES	-2.91	0.46	-2.90	0.57	-2.58	0.41	-2.90	0.57	-2.90	0.44
96*PS	-3.76	1.46	-3.76	1.42	-0.83	0.90	-3.76	1.42	-3.76	1.29
96*Genesis	-5.54	0.99	-5.53	0.99	-4.35	0.54	-5.53	0.99	-5.53	0.72
96*Saturn	-0.82	0.34	-0.82	0.33	-0.60	0.32	-0.82	0.33	-0.82	0.31
96*SNES	-2.94	0.59	-2.94	0.64	-2.63	0.46	-2.94	0.64	-2.94	0.51
96*N64	-4.63	1.28	-4.63	0.93	-2.05	0.63	-4.63	0.93	-4.63	0.78
97*PS	-2.84	1.29	-2.85	1.30	-0.70	0.80	-2.85	1.30	-2.85	1.19
97*Genesis	-4.29	0.91	-4.29	0.84	-3.51	0.50	-4.29	0.84	-4.29	0.64
97*Saturn	-0.42	0.52	-0.42	0.42	-0.65	0.35	-0.42	0.42	-0.42	0.39
97*SNES	-1.54	0.51	-1.54	0.56	-1.67	0.43	-1.54	0.56	-1.54	0.49
97*N64	-4.04	1.28	-4.03	0.82	-2.20	0.55	-4.03	0.82	-4.03	0.73
98*PS	-1.25	1.00	-1.25	1.02	0.32	0.66	-1.25	1.02	-1.25	0.92
98*Genesis	-1.90	0.44	-1.90	0.44	-1.46	0.32	-1.90	0.44	-1.90	0.37
98*SNES	0.08	0.26	0.08	0.23	-0.17	0.29	0.08	0.23	0.08	0.22
98*N64	-2.41	1.03	-2.41	0.61	-0.96	0.45	-2.41	0.61	-2.41	0.57
99*PS	-0.71	0.65	-0.71	0.70	0.41	0.49	-0.71	0.70	-0.71	0.64
99*Genesis	-0.14	0.26	-0.14	0.27	-0.14	0.27	-0.14	0.27	-0.14	0.26
99*Dream	0.17	0.67	0.17	0.59	1.04	0.45	0.17	0.59	0.17	0.50
99*N64	-1.46	0.52	-1.46	0.43	-0.48	0.37	-1.46	0.43	-1.46	0.40
00*PS	-0.64	0.38	-0.64	0.39	-0.02	0.34	-0.64	0.39	-0.64	0.37
00*PS2	-0.72	0.43	-0.72	0.43	0.28	0.45	-0.72	0.43	-0.72	0.39
00*Dream	-0.01	0.42	-0.01	0.49	0.35	0.32	-0.01	0.49	-0.01	0.41
00*N64	-0.56	0.31	-0.56	0.31	-0.18	0.30	-0.56	0.31	-0.56	0.30
PS2	1.31	0.94	1.31	0.57	0.68	0.39	1.31	0.57	1.31	0.51
Genesis	-1.28	1.54	-1.29	1.37	0.72	0.81	-1.29	1.37	-1.29	1.23
Saturn	-3.71	1.46	-3.71	1.39	-0.67	0.86	-3.71	1.39	-3.71	1.27
Dream	2.21	1.34	2.19	1.41	2.70	0.69	2.19	1.41	2.19	1.09
NES	-7.07	2.18	-7.07	1.82	-2.92	1.08	-7.07	1.82	-7.07	1.64
SNES	-3.31	1.31	-3.31	1.31	-0.46	0.81	-3.31	1.31	-3.31	1.19
N64	2.27	1.16	2.25	1.18	2.63	0.64	2.25	1.18	2.25	0.99
lambda	1.003	0.554					J-tests	13.70		
F-tests	h_price	876.4	h_price	876.4	-		h_price	876.4		
	h_soft_N	2882.2	h_soft_N	2882.2	-		h_soft_N	2882.2		
	ln(sj|B=1)	429.0	ln(sj|B=1)	429.0	-		ln(sj|B=1)	429.0		

Software Estimation

	(S1)		(S2)		(S3)		(S4)			
IB	2.19	0.10	2.94	0.20	1.47	0.04	2.30	0.12	2.94	0.20
IB_age			0.44	0.12	-0.01	0.01	0.38	0.09	0.44	0.12
h_age			-7.58	2.05	-0.24	0.23	-6.40	1.53	-7.58	2.04
PS	-30.34	1.76	-40.52	3.46	-16.63	0.88	-30.98	2.16	-40.52	3.44
PS2	-25.28	1.59	-35.60	2.94	-14.45	0.67	-26.44	1.85	-35.60	2.91
Genesis	-32.73	1.78	-41.55	4.06	-16.39	1.26	-32.30	2.72	-41.55	4.11
Saturn	-27.02	1.47	-31.44	2.54	-14.85	0.82	-23.63	2.22	-31.44	2.49
Dream	-27.52	1.53	-36.96	2.84	-16.33	0.67	-27.97	1.84	-36.96	2.80
NES	-36.80	1.83	-47.14	4.91	-18.84	1.54	-37.99	3.21	-47.14	4.98
SNES	-32.07	1.74	-40.27	3.68	-16.70	1.12	-30.88	2.52	-40.27	3.70
N64	-31.43	1.69	-41.54	3.33	-18.14	0.83	-31.89	2.06	-41.54	3.32
1994	3.95	0.19	2.67	1.48	0.91	0.62	2.22	1.21	2.67	1.48
1995	4.16	0.19	2.79	1.33	1.19	0.53	2.20	1.08	2.79	1.31
1996	3.62	0.17	2.61	1.05	1.00	0.45	2.08	0.85	2.61	1.05
1997	2.39	0.12	1.47	0.84	0.45	0.36	1.16	0.69	1.47	0.85
1998	1.63	0.10	0.82	0.65	0.29	0.28	0.64	0.53	0.82	0.65
1999	1.02	0.11	0.43	0.46	0.13	0.20	0.28	0.38	0.43	0.47
2000	0.65	0.12	0.34	0.32	0.14	0.13	0.24	0.26	0.34	0.33
F-tests	-		-		-		-		h_price	1.44E+03
	-		-		-		-		h_soft_N	4.75E+03
	-		-		-		-		ln(sj|B=1)	706.4538
	IB	4.52E+04	IB	4.52E+04	-		IB	4.24E+04	IB	2.04E+04
	-		IB_age	1.29E+05	-		IB_age	1.17E+05	IB_age	5.83E+04

FIGURE A1
Rivalry in the Other Three U.S. Video Game Systems, 1994-2002

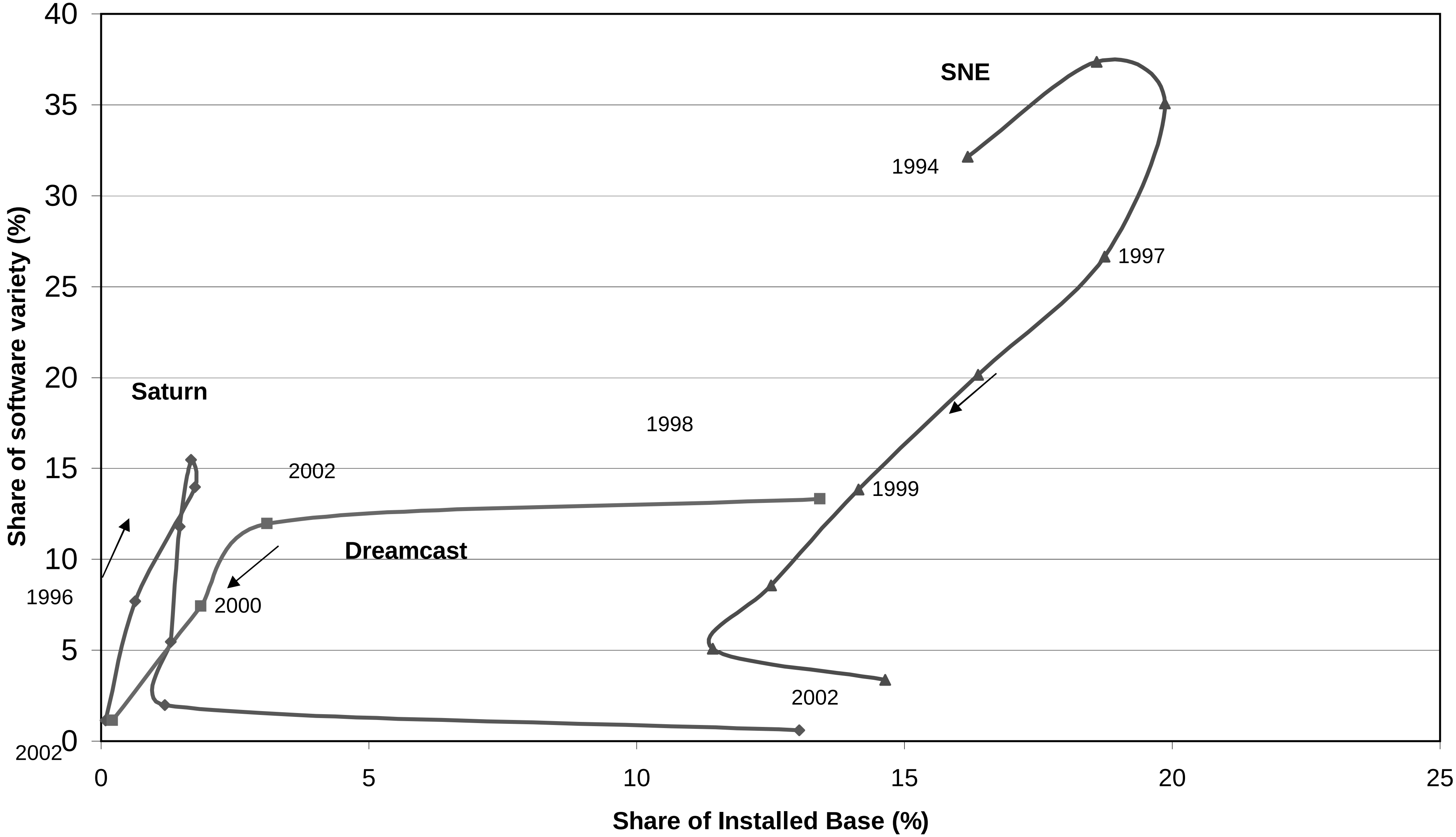

FIGURE A2
Network Effects in the Hardware Market (5) (Appendix to Figure 2)

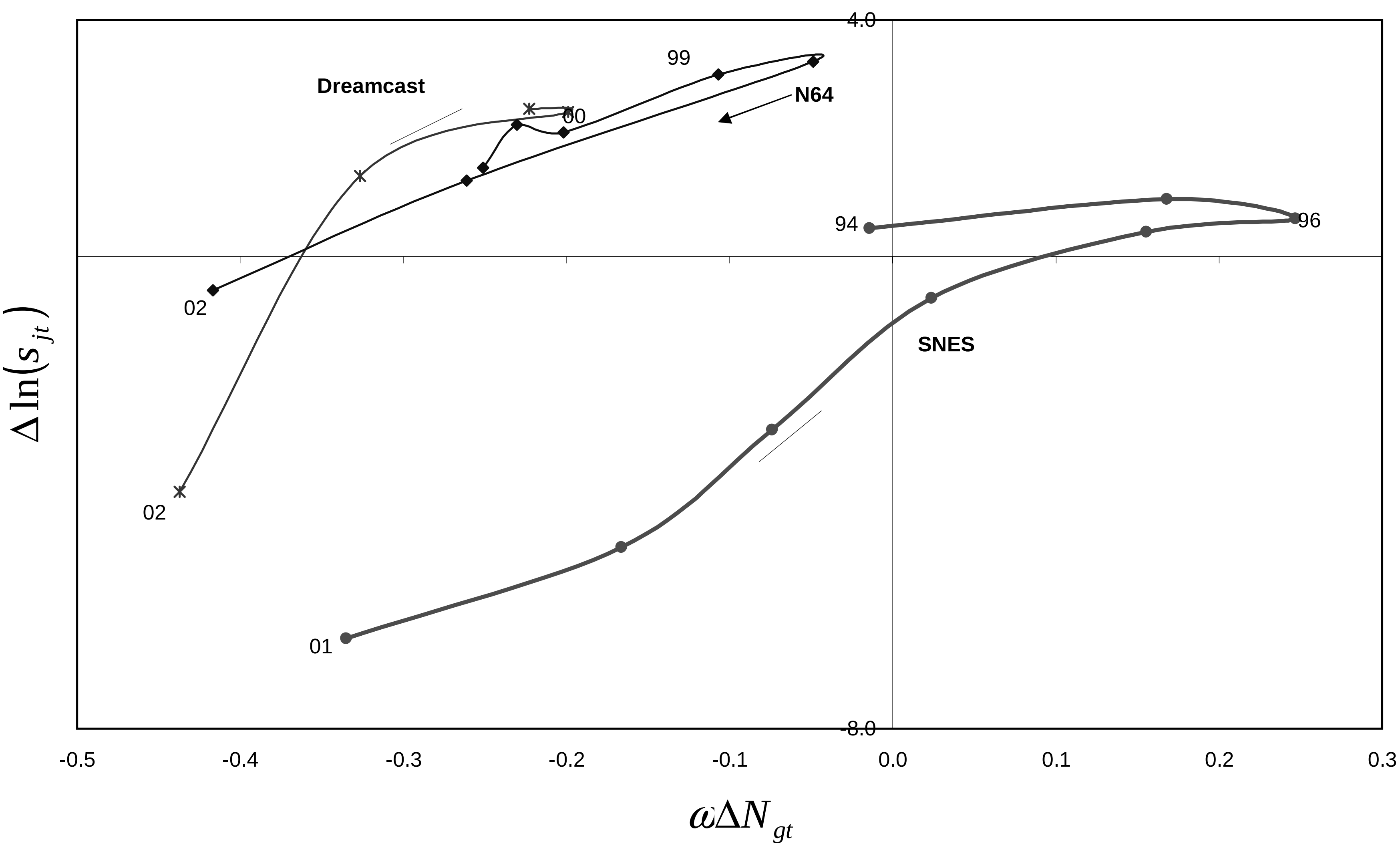

FIGURE A3
Indirect Network Effect in the Software Market (6) (Appendix to Figure 3)

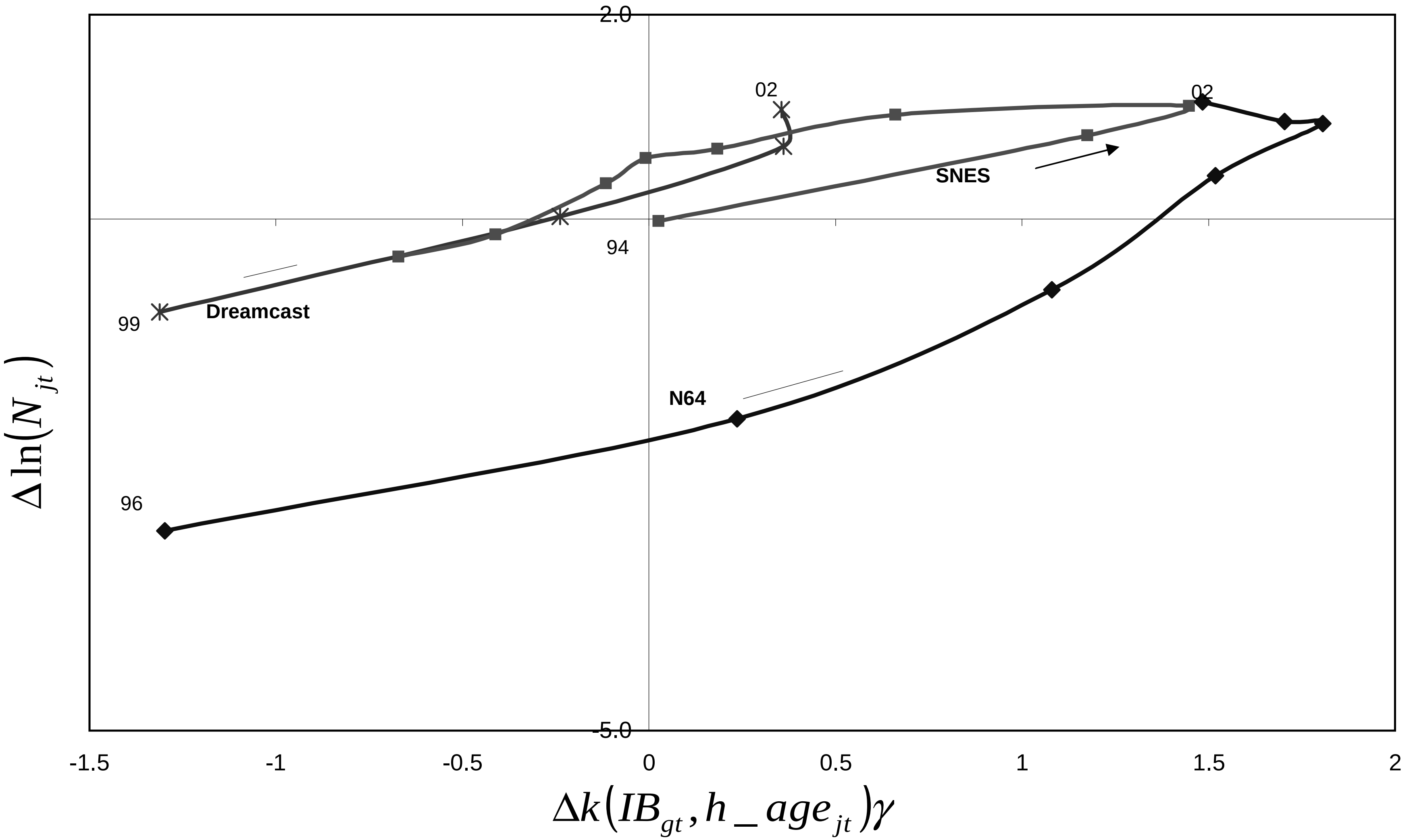

FIGURE A4
Indirect Network Effects in the Software Market for Major Consoles (6)
(without use of hardware age)

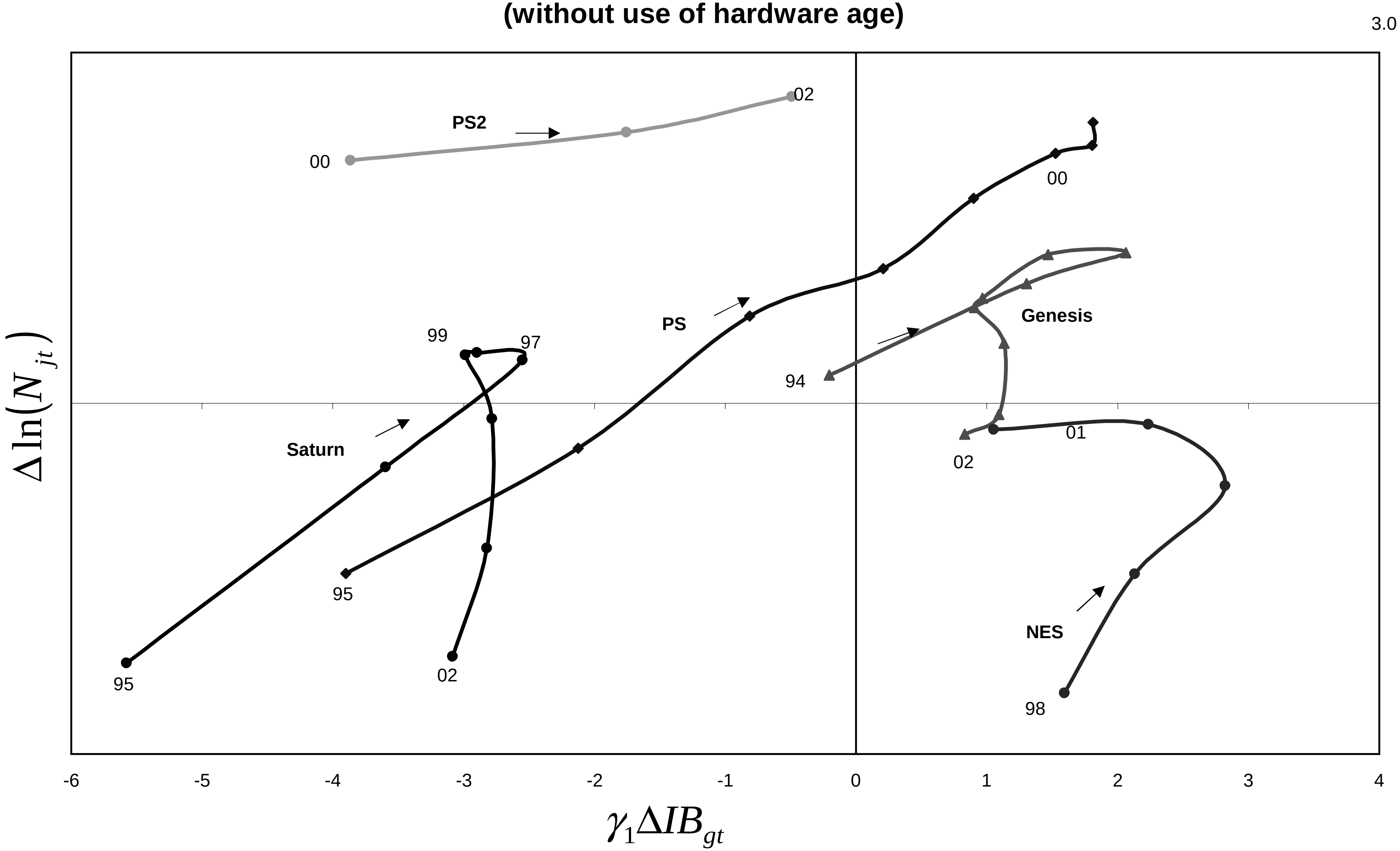

FIGURE A5
Market Share and Price
for Major Consoles Introduced after 1994