System Components, Network Effects, and Bundling[*]

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Abstract

I investigate the competition between suppliers of components of a system for which there are network effects among users. Bundling one of these components with an outside good reduces the cost to consumers of using the system. This cost reduction is not necessarily welfare-enhancing, and bundling can also reduce welfare by decreasing innovation incentives. The model is used to evaluate Microsoft's bundling of Windows with Internet Explorer and its effect on competition with Netscape.

1 Introduction

One allegation in the recent antitrust suit against Microsoft is that the bundling of Internet Explorer with the Windows operating system is anticompetitive.[1] It is not clear whether this practice is likely to enhance social welfare, due to several unique aspects of the market for Internet browsers. One such aspect is the presence of network effects, which can complicate traditional welfare analysis. Another is that the browser itself can be viewed as one component of a system with the unusual characteristic that each component is purchased by a different party. Consumers use the browser to view web content, and Internet content providers (ICPs) use associated software to create this content.[2] Browser suppliers license this software to ICPs that would like to attract consumers to their websites. The browser and the complementary software together comprise a system that is of value to the consumer, but the consumer only buys one component of the system.[3] In fact, the browser may be offered to consumers for free, in order to stimulate demand for the other component.[4] Given that the two principal competitors in the browser market, Microsoft and Netscape, do offer their browsers for free, the effect of bundling one of the browsers with another good is unclear.

Furthermore, one effect of bundling is to reduce the cost to the consumer of using the browser. If Internet Explorer is bundled with Windows, the user faces a lower

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[1] United States v. Microsoft, Civil Action No. 98-1232. The Court's Findings of Fact, released on November 5, 1999, can be found at www.usdoj.gov/atr/cases/f3800/msjudgex.htm; the Court's Conclusions of Law, released on April 3, 2000, can be found at www.usdoj.gov/atr/cases/f4400/4469.htm.

[2] HTML, the language used to display text and simple graphics on the web, is non-proprietary, but much of the more sophisticated multimedia software is proprietary.

[3] One might also consider one component to be a partially disabled version of the other component, as in the case of Adobe Acrobat and Acrobat Reader. Hahn (2001) models components in this manner.

[4] Another source of revenue for a browser supplier is the sale of network management tools, which are browser-specific, to institutional users of the browser.

cost of installing the software or learning how to use it. In addition, Internet Explorer is more seamlessly integrated into Windows. Since non-Microsoft developers do not have access to all the code for Windows, products like Netscape Navigator cannot function as well with Windows. This cost reduction can actually lower welfare under some circumstances.[5]

Previous models have offered insight into the effects of bundling, but none are easily applicable to this market. Whinston (1990) shows that there are situations in which it is possible and profitable for a firm with a monopoly in one good to foreclose competition in the market for a second good by tying[6] the two products, and that social welfare may be higher without tying. McAfee, McMillan, and Whinston (1989) explore the use of bundling as a means of price discrimination. Nalebuff (1999) finds that the gains to a firm from the price-discrimination effect of bundling are small compared to the gains from the foreclosure effect. Carlton and Waldman (2002) investigate the use of tying to preserve monopoly power through intertemporal economies of scope.

Other work has illuminated various aspects of the browser market and the effect of bundling: Economides (1998) finds that a firm in Microsoft's position has incentive to degrade the quality of the monopolized good when it is combined with competitors' products, and Choi and Stefanadis (2001) consider the effect of bundling on the investment incentives of potential entrants. However, no single model has taken into account all of the idiosyncratic characteristics of the browser market that have a bearing on the Microsoft case. It is the purpose of this paper to provide such a

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[5]The only effect of bundling in this paper is the cost reduction; apart from the specific analysis of the consequences of Microsoft's bundling of Internet Explorer with Windows, this paper can be viewed as an analysis of cost reductions in a setting of network effects. Bundling is simply one example of such a cost reduction.

[6]The literature has made a distinction between *tying* and *bundling*: tying is the requirement that a consumer who is buying one good also buy another from the same firm, while bundling is the requirement that consumers buy two goods in fixed proportions. Strictly speaking, this paper considers tying, but the distinction is not meaningful in this context.

model. The model also has broader implications; there are other systems, such as Adobe Acrobat and Real Player, that are comprised of components purchased by different parties.[7] The reasoning used here could also illuminate aspects of these markets.

In Section 2, I model the market for Internet browsers, and I discuss the differences from previous models. In Section 3, I solve for equilibrium prices and market shares with and without bundling. In Section 4, I characterize the conditions under which firms will offer one component for free. I consider the welfare effects of bundling in Section 5. In Section 6, I extend the model to consider firms' incentives to invest in the development of system components. In Section 7, I conclude by relating these results to the Microsoft case. The cost-reducing effect of bundling has ambiguous welfare effects, and the ability of Microsoft to bundle other software products with Windows discourages other software firms from investing in new products. In the long term, even if there are gains from reduction of user costs, the losses from stifled investment may outweigh them. The same effect would be expected, and the same concerns would arise, in other markets for system components.

2 The model

There are two firms, 1 and 2, which each offer two components of a system, A (sold to consumers) and B (sold to ICPs);[8] firm i supplies A_i and B_i. A system is composed of A and B, and each firm's components are incompatible with those of the other firm.

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[7]Adobe Acrobat is used to create pdf documents; Acrobat Reader is used to view these documents. Real Player is used to view or listen to content that has been created by separate software used to create this content.

[8]I will continue to refer to consumers of good B as "ICPs" and consumers of good A simply as "consumers."

The two firms' products are maximally differentiated in two distinct linear product spaces: firm 1's components are located at 0, firm 2's at 1. All components have a marginal cost of zero. If firm i sells N_{A_i} units of A_i at price p_{A_i} and N_{B_i} units of B_i at price p_{B_i}, its profit is $\pi_i = N_{A_i} p_{A_i} + N_{B_i} p_{B_i}$. Firm 1 also has a monopoly on a third good, W, that it may bundle with A_1 (I discuss the effects of this bundling below). All consumers have a common reservation price for W, p_w. Without bundling, firm 1 sets the price of W equal to p_w and sells to all consumers, earning a profit of $\pi_w = (p_w - c_w)$, where c_w is the marginal cost of W. That is, all consumers purchase W, no matter which version of A they buy; since all consumers buy at the reservation price, they obtain no net utility from W itself. The profit that firm 1 obtains from W does not affect its behavior in the markets for A and B; therefore, π_w is omitted from the analysis that follows.[9]

Consumers' preferences regarding A_1 and A_2 are described by x, which is uniformly distributed from 0 to 1. ICPs' preferences regarding B_1 and B_2 are similarly described by y. The total mass of consumers is one, as is the mass of ICPs.[10] Different firms' components may be of different qualities: disregarding network effects and travel costs, a consumer's utility for A_i is U_{A_i}, and similarly for an ICP's utility[11] for B_i.

There is an indirect network effect between consumers and ICPs: the value of A_i to consumers is strictly increasing in the number of ICPs that use B_i, and the value of

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[9] I assume that what happens in the markets for A and B does not affect the market for W. This is justified if the markets for A and B are relatively small. The operating system market is likely to influence the browser market, but the reverse is not true. It is then reasonable to take aspects of the market for W to be exogenous.

[10] Allowing for a difference between the mass of consumers and the mass of ICPs does not qualitatively change the analysis. If, for example, the mass of ICPs were greater, we would see such effects as quality changes of component B having a relatively greater impact on prices. The comparative statics and the qualitative welfare results would remain the same.

[11] Although ICPs are likely to be profit-maximizing firms, I consider their objective to be a utility in order to be as general as possible. ICPs may profit directly from consumers' visiting their sites; or they may profit indirectly, as through advertising revenue; or they may benefit in some other way. An ICP may simply wish to disseminate information as widely as possible. The key point is that an ICP benefits from additional consumers' viewing its content.

B_i to ICPs is strictly increasing in the number of consumers that use A_i. In choosing a browser, a consumer doesn't care how many other consumers are using the same browser, but does care how much content is available for that browser. Similarly, ICPs only care about how many consumers will be able to view their content.

Some web content can be viewed equally well with either browser. There is, however, a great deal of content that is only viewable with one browser, or is more easily viewable with one browser. One often encounters icons on web pages marking the spot where an image would appear if the page were being viewed with another browser; and many web pages note that they are optimally viewed with a particular browser. This has changed somewhat over time but was certainly true in the early- to mid-1990s, the time period under scrutiny in the Microsoft case. If there is some degree of compatibility between systems, it can be thought of as weakening the network effect.[12]

The constants m and n quantify the magnitudes of the two network effects: m is the marginal value to a consumer using A_i of an additional ICP using B_i, and n is the marginal value to an ICP using B_i of an additional consumer using A_i.[13]

In addition to the price, a consumer located at x that buys A_1 incurs cost tx; for A_2, the cost is $t(1-x)$.[14] I use x^* to denote the location of the consumer indifferent between A_1 and A_2; then x^* will also be the proportion of consumers that buy A_1. The cost for an ICP is ty or $t(1-y)$, and y^* is the location of the indifferent ICP and the proportion of ICPs that buy B_1.

The setup cost to the user, which may reflect installation or learning costs, or

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[12]If some consumers use both browsers, this can also be thought of as weakening the network effect. Information about inter-browser compatibility issues can be found at www.w3.org.

[13]I am assuming a constant marginal benefit to network size. All of the results in the paper would hold if this marginal benefit were decreasing.

[14]This "travel cost" is not meant to be literal. One can think of the distance between the goods, x, as the degree of differentiation in some characteristic. The unit cost, t, reflects the strength of consumers' preferences for one good over the other, and the total cost, tx or $t(1-x)$, reflects the cost to the consumer of using a less-than-ideal good.

difficulties of using a component that is *not* bundled with W (because, for example, an unbundled browser crashes more often) is c.[15] The setup cost is the same for A_1 and A_2 in the no-bundling case. This cost can be eliminated by integrating A with W: if firm 1 bundles the two goods, consumers that purchase A_1 will not incur c, but consumers that purchase A_2 will. ICPs do not incur a setup cost.

Let δ_b be equal to 0 when there is no bundling and 1 when A_1 is bundled with W. The utility of a consumer located at x is $U_{A_1} + my^* - p_{A_1} - tx - (1 - \delta_b)c$ if the consumer buys A_1, and $U_{A_2} + m(1 - y^*) - p_{A_2} - t(1 - x) - c$ if the consumer buys A_2. The utility of an ICP located at y is $U_{B_1} + nx^* - p_{B_1} - ty$ if the ICP purchases B_1 and $U_{B_2} + n(1 - x^*) - p_{B_2} - t(1 - y)$ if the ICP purchases B_2. Let $\Delta U_A = U_{A_2} - U_{A_1}$ and $\Delta U_B = U_{B_2} - U_{B_1}$: there may be vertical (quality) differences between the components as well as horizontal (taste) differences. I assume that the base utilities are at least as great as the non-price costs: $U_{A_1}, U_{A_2} > t + c$ and $U_{B_1}, U_{B_2} > t$. Given this, there exist strictly positive prices at which all consumers and ICPs will buy one of the components.

The crucial differences between this model and other models relating to the Microsoft case are the specification of the indirect network effect and the effect of bundling on the user's setup cost.

In the first stage, firms set prices. In the second stage, consumers and ICPs choose which components they will buy. In equilibrium, firms, consumers and ICPs correctly anticipate demand for each component.

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[15]This cost is similar to Farrell and Gallini's (1988) setup costs, or Farrell and Shapiro's (1988) switching costs.

3   Equilibrium prices and market shares

I focus on the conditions under which the interior equilibrium is unique: both firms have positive sales ($0 < x^* < 1$ and $0 < y^* < 1$). If network effects or quality differences are large enough, it is possible that one firm is foreclosed from one or both markets. These equilibria are characterized in the appendix, and some of these cases are discussed in Section 5.3. All proofs are in the appendix.

It is straightforward to derive equilibrium prices[16] from firms' profit maximization. The sufficient conditions for the uniqueness of this equilibrium essentially say that the strength of consumer preference is larger than either of the network effects, that differences in quality are not too large, and that the setup cost is not too large.

Proposition 1 *Given*

$(A1)$ $t > m, \ t > n$

$(A2)$ $m - t + c < \Delta U_A < t - m$

$(A3)$ $n - t < \Delta U_B < t - n$

$(A4)$ $c < 2(t - m)$

the unique equilibrium prices are

$$p^*_{A_1} = t - n + \frac{(\delta_b c - \Delta U_A)(3t^2 - 2mn - n^2) + \Delta U_B (n - m) t}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{1}$$

$$p^*_{A_2} = t - n - \frac{(\delta_b c - \Delta U_A)(3t^2 - 2mn - n^2) + \Delta U_B (n - m) t}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{2}$$

$$p^*_{B_1} = t - m + \frac{(\delta_b c - \Delta U_A)(n - m) t - \Delta U_B (3t^2 - 2mn - m^2)}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{3}$$

$$p^*_{B_2} = t - m - \frac{(\delta_b c - \Delta U_A)(n - m) t - \Delta U_B (3t^2 - 2mn - n^2)}{9t^2 - 2n^2 - 5mn - 2m^2}. \tag{4}$$

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[16]When there is bundling, I consider the "price" of A_1 to be the difference between the price of the bundle (W and A_1) and the reservation price of W. I use p_{A_1} to refer both to the actual price of A_1 without bundling and to the fictitious price of A_1 with bundling.

Market shares of firm 1 are

$$x^* = \frac{1}{2} + \frac{\left(p_{A_2}^* - p_{A_1}^* - \Delta U_A + \delta_b c\right) t + \left(p_{B_2}^* - p_{B_1}^* - \Delta U_B\right) m}{2\left(t^2 - mn\right)} \qquad (5)$$

$$y^* = \frac{1}{2} + \frac{\left(p_{B_2}^* - p_{B_1}^* - \Delta U_B\right) t + \left(p_{A_2}^* - p_{A_1}^* - \Delta U_A + \delta_b c\right) n}{2\left(t^2 - mn\right)} \qquad (6)$$

and $0 < x^*, y^* < 1$.

It is possible for there to be an interior equilibrium when not all of $(A1) - (A4)$ hold. See, in particular, Section 4. The appendix details all of the cases in which one firm is foreclosed from one or both markets—the cases in which at least one of x^* and y^* is equal to 0 or 1. Assumptions $(A1) - (A4)$ guarantee that bundling will not move the market from an interior equilibrium to an equilibrium in which one or both markets are tipped.

Proposition 2 *Given $(A1) - (A4)$, bundling:*

    (i) increases $p_{A_1}^*$;

    (ii) decreases $p_{A_2}^*$;

    (iii) increases x^* and y^*;

    (iv) increases π_1; and

    (v) decreases π_2.

    If $n > m$, bundling increases $p_{B_1}^*$ and decreases $p_{B_2}^*$. If $m > n$, bundling decreases $p_{B_1}^*$ and increases $p_{B_2}^*$.

Not all of the comparative statics of the two-component system are intuitive. When there is bundling, the changes in the prices of B may or may not be in the same direction as the prices of A. By raising the price of B, a firm profits directly from sales of B; but by lowering the price of B, a firm profits by encouraging sales of A. When $n > m$ (the marginal value to ICPs of an additional consumer on the same

network is greater than the marginal value to consumers of an additional ICP), the first effect dominates, and p_{B_1} increases. When $m > n$, the second effect dominates, and p_{B_1} decreases. The price of B_2 always moves in the opposite direction of the price of B_1.

Changes in the difference in quality of the goods have similar effects on prices. As ΔU_A increases (A_2 becomes more valuable relative to A_1), $p_{A_1}^*$ falls and $p_{A_2}^*$ rises; $p_{B_1}^*$ will fall and $p_{B_2}^*$ will rise if $n > m$, and $p_{B_1}^*$ will rise and $p_{B_2}^*$ will fall if $m > n$. Changes in ΔU_B have an analogous effect.

4 One free component

If network effects are strong enough, firms may find it profitable to give one component away to increase profits from the other component. Consider the prices for A when there are no quality differences and no bundling: $p_{A_1} = t - n$ and $p_{A_2} = t - n$. These prices are negative if $n > t$; this means that the marginal value to an ICP of an additional consumer on the same network is greater than the travel cost. In this situation, firms want to lower the price of A as much as possible to attract consumers to their network. This will enable firms to charge the ICPs higher prices for B. A similar situation arises if $m > t$: firms offer B to ICPs for free and charge consumers more for A.[17] In general, p_{A_1} and p_{A_2} will be zero if n is large enough.

Proposition 3 *For n sufficiently large, $p_{A_1} = p_{A_2} = 0$. For m sufficiently large, $p_{B_1} = p_{B_2} = 0$.*

If either m or n is very large, it is possible for one market to tip (although the interior equilibrium also exists). If m and n are both large enough that the expres-

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[17]If both of these conditions were true, both goods would be priced at the marginal cost of zero, and equilibria may exist in which all consumers and ICPs choose one network or the other. This case is described in the appendix.

sions for all four prices are negative, it is likely that both markets will tip—for very large network effects, consumers and ICPs will have a strong preference for one large network. I consider these cases in Section 5.3.[18]

It is again straightforward to solve for the prices for B when n is large, and to specify sufficient conditions for these prices to be an equilibrium.

Proposition 4 *Given*

    $(A1')$ $t^2 > mn$

    $(A2')$ $m - t + c < \Delta U_A < t - m$

    $(A3')$ $t - n < \Delta U_B < n - t$

    $(A4')$ $c < 2(t - m)$

and n large, the unique equilibrium is

$$p^*_{A_1} = p^*_{A_2} = 0 \tag{7}$$

$$p^*_{B_1} = t - \frac{mn}{t} - \frac{\Delta U_B}{3} + \frac{n(\delta_b c - \Delta U_A)}{3t} \tag{8}$$

$$p^*_{B_2} = t - \frac{mn}{t} + \frac{\Delta U_B}{3} - \frac{n(\delta_b c - \Delta U_A)}{3t}. \tag{9}$$

Now all the effects of bundling and quality changes are unambiguous. Bundling increases the price of the component with the nonzero price for the bundling firm, and decreases this price for the other firm. Changes in the quality of the components have the expected effects.

Corollary 1 *Given $(A1') - (A4')$ and n large, bundling*

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[18]Unless there are no quality differences and no bundling, as n increases, one price goes to zero before the other; there is some n for which one of $p^*_{A_1}$ and $p^*_{A_2}$ is zero and the other is positive (which is which depends on the difference in quality and whether firm 1 is bundling). This case offers no additional insight; in what follows, I consider equilibria in which either both components are free or neither are.

(*i*) *increases* $p^*_{B_1}$;

(*ii*) *decreases* $p^*_{B_2}$; *and*

(*iii*) *increases both* x^* *and* y^*.

Increasing ΔU_A *or* ΔU_B *increases* $p^*_{B_2}$ *and decreases* $p^*_{B_1}$.

5   Welfare

I take social welfare to be the sum of consumer utility, ICP utility, and industry profit. The total utility of consumers, CU, is the sum of the utilities of consumers that buy A_1 and consumers that buy A_2 :

$$
\begin{aligned}
CU &= \int_0^{x^*} (U_{A_1} + my^* - tx - p_{A_1} - \delta_{nb}c)dx \\
&\quad + \int_{x^*}^1 [U_{A_2} + m(1 - y^*) - t(1 - x) - p_{A_2} - c]dx.
\end{aligned}
\tag{10}
$$

Similarly, the total utility of ICPs, $ICPU$, is

$$
ICPU = \int_0^{y^*} (U_{B_1} + nx^* - p_{B_1} - ty)dy + \int_{y^*}^1 [U_{B_2} + n(1 - x^*) - p_{B_2} - t(1 - y)]dy. \tag{11}
$$

Total industry profit is

$$
\pi_1 + \pi_2 = p_{A_1}x^* + p_{B_1}y^* + p_{A_2}(1 - x^*) + p_{B_2}(1 - y^*). \tag{12}
$$

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Total welfare is then

$$
\begin{aligned}
W \;=\; & U_{A_2} + U_{B_2} - \Delta U_A x^* - \Delta U_B y^* \\
& + (m+n)\left(2x^* y^* - x^* - y^* + 1\right) + t\left(-x^{*2} - y^{*2} + x^* + y^* - 1\right) \\
& + c\left(\delta_b x^* - 1\right).
\end{aligned}
\tag{13}
$$

Note that we can write welfare in terms of x^* and y^*; prices only influence welfare through their influence on x^* and y^*. The prices themselves represent a welfare-neutral transfer.

To begin examining the difference in welfare induced by bundling, I first consider some special cases.

5.1   Equal qualities

If components are of the same quality $(\Delta U_A = \Delta U_B = 0)$, then comparison of welfare with and without bundling is straightforward. In this case, bundling increases welfare.

Proposition 5 *When goods are of equal quality $(\Delta U_A = \Delta U_B = 0)$ and $(A1) - (A4)$ hold, social welfare is higher with bundling than without: $W_b > W_{nb}$.*

Let $\Delta W = W_b - W_{nb}$ be the gain in welfare from bundling. This gain is greater for greater values of c, m, or n. The larger gain in welfare coincides with firm 1 taking a larger share of the market.

Proposition 6 *When goods are of equal quality $(\Delta U_A = \Delta U_B = 0)$ and $(A1) - (A4)$ hold, ΔW decreases with t and increases with c, m, and n.*

For greater values of t, bundling has a relatively lesser effect on welfare; for greater values of m or n, bundling has a relatively greater effect.

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5.2 No network effects

When there are no network effects ($m = n = 0$), comparison of welfare with and without bundling is again straightforward. However, without the assumption of equal qualities, it is possible that bundling lowers welfare. I assume here that $\Delta U_B = 0$; this does not affect the analysis of the impact of bundling in the market for A.

Proposition 7 *When there are no network effects ($m = n = 0$) and $(A1) - (A4)$ hold, bundling reduces welfare if and only if $\Delta U_A > \frac{18t+5c}{7}$; i.e., if A_2 is sufficiently superior to A_1.*

If the above condition holds, welfare is maximized when all consumers buy the higher quality good. Firm 2 is able to set a price such that profit will be positive and all consumers will buy A_2, but this does not maximize firm 2's profit. The high quality of A_2 creates an opportunity for surplus in the market, but firm 2 is unable to capture all of this surplus and thus does not price in such a way as to maximize total surplus. The small cost reduction obtained through bundling exacerbates this inefficiency. With a lower total cost, more users will buy A_1, when it is still the case that welfare would be maximized if all consumers bought A_2.[19]

A somewhat different scenario may aid the intuition. Consider firm 2 to be a monopolist in the market for A, located at $x = 1$, with consumers uniformly distributed on $[0, 1]$. If A_2 is of very high quality, it will be in firm 2's interest to cover the entire market. Consumers will be willing to pay a high price for A_2, even if they must also incur the maximum travel cost. Firm 2 sets a price such that the consumer located at 0 is indifferent between buying and not buying. Then firm 1 enters the market at $x = 0$. A_1 is of much lower quality than A_2, but not so low that firm 1 is unable to

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[19]This is very similar to a result of Schwartz (1989): If one firm in an oligopoly achieves a cost reduction through (potentially costless) investment, welfare can decrease because of an inefficient reshu- ing of output.

take any of the market away from firm 2. Firm 2 does not want to price aggressively enough to hold on to the entire market, although it could. A small minority of consumers buy A_1, although total welfare would be higher if all consumers bought A_2. Firm 2 has a great deal of market power, but when firm 2 is a monopolist, total surplus is maximized (with firm 2 taking most of the surplus) because the entire market is covered. Welfare decreases with the entrance of firm 1, because then firms' pricing incentives distort the allocation of A_1 and A_2 away from the social optimum. This inefficiency increases for small increases in firm 1's quality, small decreases in firm 1's marginal cost, or small decreases in the setup cost of A_1.[20]

If ΔU_A is indeed very high, it may be that firm 2 takes the entire market whether or not firm 1 bundles. The effect on welfare in this case is discussed below. But it is also possible for the above condition to hold with interior solutions for the bundling and non-bundling cases, as the following example illustrates.

Example 1 *Let $c = 1$, $t = 5$, $m = n = 0$, $\Delta U_A = 14$, and $\Delta U_B = 0$. Then x^* is 0.0333 without bundling and 0.0667 with bundling, y^* is .5 with or without bundling, and non-bundling welfare is greater by 0.21667.*

For this set of parameters, the sufficient condition for firm 2 to take all of the market for A is satisfied. However, to do this firm 2 must lower p_{A_1} enough that the loss of revenue outweighs the gain in market share. Firm 2 profits more by allowing firm 1 to take a small share of the market.

5.3 Tipped-market equilibria

Next I consider welfare when qualities are unequal and network effects are strong enough that at least one of the markets (A or B) is tipped. That is, at least one of x^*

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[20]This assumes that firm 2 must set only one price. If firm 2 can price discriminate, then when firm 1 enters, firm 2 will offer a lower price to those consumers closest to 0, and firm 1 will have no sales.

and y^* is equal to zero or one in equilibrium. It is possible for one market to be split and the other to tip, or for both markets to tip in favor of the same firm, or even for each market to tip in a different firm's favor. All of these cases are described in the proof of Proposition 1 in the appendix. In this section, I will only discuss the cases in which both markets split or both markets tip in favor of the same firm. All of the relevant welfare results can be seen in these cases; there are no qualitative differences in the omitted cases.

There are configurations of parameters for which more than one of the following equilibria exist. In addition, the interior equilibrium described in Section 3 always exists. I am not intending to argue what the market outcome will be for any particular set of parameters; I am simply examining the welfare consequences of bundling *if* the actual outcome is one of these cases.

Each of these equilibria has sufficient conditions associated with it, described in the appendix. Each set of conditions sets bounds on the parameters such that one firm may monopolize a market at a nonnegative price and no consumer or ICP will want to switch to the other firm.

(i) **Firm 1 monopolizes both markets $(x^* = y^* = 1)$ with and without bundling:** The effect of bundling is that consumers' price increases by c, but their cost decreases by c, and that firm 1's profit increases by c. Bundling increases total welfare by c.

(ii) **Firm 2 monopolizes both markets $(x^* = y^* = 0)$ with and without bundling:** The only effect of bundling is that firm 2 must lower its price (against threat of entry by firm 1). Consumers gain by the amount of firm 2's loss, and bundling is welfare-neutral.

(iii) **Firm 2 monopolizes both markets without bundling, firm 1 monopolizes both markets with bundling:** The effects of bundling are the following:

ICPs' price decreases by ΔU_B, and the quality of their good decreases by ΔU_B; consumers' price increases by $c - \Delta U_A$, their cost decreases by c, and the quality of their good decreases by ΔU_A. The changes in firms' profits are welfare-neutral transfers. Bundling causes a net gain in total welfare of c.

(iv) **Split markets without bundling, firm 1 monopolizes both markets with bundling:**   Welfare decreases under bundling if and only if

$$-\Delta U_A - \Delta U_B + \Delta U_A x^* + \Delta U_B y^*$$
$$- (m + n)\left(2x^* y^* - x^* - y^*\right) - t\left(-x^{*2} - y^{*2} + x^* + y^*\right) + c < 0. \qquad (14)$$

This condition holds if x^* and y^* are very small and ΔU_A and ΔU_B are large. However, the sufficient conditions for the existence of this equilibrium are $m - t + c - \Delta U_A \geq 0$ and $n - t - \Delta U_B \geq 0$; i.e., network effects are large relative to quality differences. This means that $(m + n)\left(2x^* y^* - x^* - y^*\right)$ can be large even if x^* and y^* are small. This acts in favor of bundling being welfare enhancing. If bundling does reduce welfare, the scenario is similar to that in Proposition 7: bundling is welfare-reducing because of firm 2's superior quality and firm 1's tiny market share.

(v) **Firm 2 monopolizes both markets without bundling, split markets with bundling:**   Bundling lowers welfare if and only if

$$-\Delta U_A x^* - \Delta U_B y^* + (m + n)\left(2x^* y^* - x^* - y^*\right)$$

$$+ t\left(-x^{*2} - y^{*2} + x^* + y^*\right) + cx^* < 0. \qquad (15)$$

The sufficient conditions for this equilibrium are $m - t + \Delta U_A \geq 0$ and $n - t - \Delta U_B \geq 0$. Given these conditions, it is unlikely that bundling will reduce welfare, as in the

preceding case. If bundling does reduce welfare, it is again for the same reasons as in Proposition 7: firm 2 has a far superior quality and firm 1 has a tiny market share.

5.4   General welfare results

Having considered some special cases, I turn now to the case of interest: when qualities are unequal and there are positive network effects, but neither market is tipped. It is clear that, in the absence of large quality differences, bundling enhances welfare. As the following proposition and example illustrate, it is possible for bundling to reduce welfare when there are large quality differences, even if network effects are present.

The magnitude of network effects has no systematic effect on the difference in welfare caused by bundling. Rather, network effects tend to magnify the effects of other parameters. If network effects are very strong, bundling enhances welfare. Strong network effects magnify firm 1's gain in market share from bundling; for a sufficiently large gain in market share, welfare increases.

Furthermore, whether the price of one good is zero has no direct bearing on the welfare effect of bundling. Bundling reduces welfare when firm 2 has a very large proportion of the market and bundling increases firm 1's market share very little. The prices that correspond to these market shares are not relevant.

Proposition 8 *If ΔU_A or ΔU_B is large relative to t, m, n, and c, and $(A1) - (A4)$ hold, bundling reduces welfare.*

Note that bundling a component lowers welfare if either the bundled component itself (A) or the *other* component (B) is sufficiently inferior. The bundling of the browser harms welfare if there is enough of a quality difference in the software used by either consumers or ICPs. If Internet Explorer or its complementary software were sufficiently inferior to Netscape's corresponding products, bundling Internet Explorer with Windows would reduce welfare, irrespective of innovation incentives.

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The following corollary follows immediately from the proof of Proposition 8.

Corollary 2 *The change in welfare due to bundling increases with ΔU_A and ΔU_B.*

It is possible that very large quality differences will cause the market to tip in firm 2's favor; but there is still an interior equilibrium in which bundling reduces welfare.

Example 2 *Let $c = 1$, $t = 5$, $m = n = .1$, $\Delta U_A = 14$, and $\Delta U_B = 0$. Then there exists an equilibrium in which x^* is* 0.0331 *without bundling and* 0.0665 *with bundling, y^* is .4907 without bundling and .4913 with bundling, and non-bundling welfare is greater by* 0.21719.

In this example, bundling causes a greater loss of welfare than in Example 1 above: the greater network effects magnify the loss in welfare. However, this is a local effect; as m and n become much larger, the qualitative impact of bundling changes.

The previous subsection discussed welfare effects when one or both markets tip. We saw there that the effect of bundling on a tipped-market equilibrium is qualitatively the same as in the interior equilibrium. On the whole, bundling is usually (weakly) welfare enhancing, but bundling lowers welfare if firm 2's components are markedly superior to firm 1's components.

6 Investment incentives

6.1 New product introduction

Now suppose that A and B do not yet exist. At stage 0, before the pricing game, firm 2 can invest in R&D to try to develop the components.[21] If firm 2 invests R, it

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[21] Choi and Stefanadis (2001) also examine the effect of bundling on a potential entrant's investment incentives. However, the reduction of the setup cost is not a factor in their model, and their results are not related to the quality of the bundled components

has probability $p(R)$ of success, where $p' > 0$ and $p'' < 0$. If firm 2 is successful, firm 1 can imitate the components, but firm 1's components will be inferior: $\Delta U_A > 0$ and $\Delta U_B > 0$. Let R_b and R_{nb} be the respective investments by firm 2 with and without bundling. Bundling does stifle innovation, in the sense that firm 2 invests less in developing a component if firm 1 has the ability to bundle that component with a monopolized product.

Proposition 9 *Firm 2 invests less in new product development if firm 1 can bundle an imitation of the new product:*

$$R_b < R_{nb}. \tag{16}$$

6.2 Welfare effects

If the components are not developed, no welfare is gained. Let W_b and W_{nb} be the respective welfare gains if the components are developed and there is or is not bundling. In both cases, taking the presence or absence of bundling as fixed, firm 2 invests less in R&D than is socially optimal, because the firm cannot appropriate all the benefits of the R&D. Letting R_b^* and R_{nb}^* be the socially optimal investments with and without bundling, $R_b < R_b^*$ and $R_{nb} < R_{nb}^*$. However, the bundling itself may either mitigate or exacerbate the inefficiency in investment.

Proposition 10 *If bundling increases post-investment welfare ($W_b > W_{nb}$), then firm 2's investment in R&D is further from the social optimum with bundling. If bundling decreases post-investment welfare ($W_{nb} > W_b$), then firm 2's investment in R&D may be (but is not necessarily) closer to the social optimum with bundling.*

Corollary 3 *If the innovation-stifling effect of bundling lowers welfare, the harm to welfare is greater if the quality difference is lower.*

By Corollary 2, for a larger quality difference, $W_b - W_{nb}$ is larger. This implies that $R_b^* - R_b$ is larger. If the imitation components are close in quality to the originals, the harm to welfare is the greatest. While the existence of high-quality components, imitation or otherwise, is beneficial, the bundling *itself* is more detrimental. But even for grossly inferior imitations, bundling stifles innovation. Even if the markets for A and B are tipped in firm 2's favor with or without bundling, bundling lowers firm 2's profit and thus induces firm 2 to invest less.

7   Conclusion: bundling and platform building

U.S. District Judge Thomas Penfield Jackson has found that "Microsoft enjoys monopoly power" in the operating system market.[22] Given this monopoly power, Microsoft can bundle Internet Explorer with Windows and thereby induce more consumers to use Internet Explorer. This profits Microsoft by increasing the demand for software complementary to Internet Explorer.

Bundling reduces the cost to the consumer of using the browser. This paper has shown that, even if we disregard investment incentives, this cost-reducing effect can lower welfare. If Netscape Navigator were of much higher quality than Internet Explorer and network effects were not too strong, Microsoft's bundling of Internet Explorer with Windows could harm welfare. It would be difficult to argue that the current version of Internet Explorer is vastly inferior to the current version of Netscape Navigator; but it would be much easier to make this argument about the early-90s versions of the products. When Microsoft began bundling, although it created a cost reduction, it might well have lowered welfare.

Bundling also discourages innovation by Microsoft's competitors and potential

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[22]Findings of Fact, Civil Action No. 98-1232, paragraph 33. The entire text of the Findings of Fact, released on November 5, 1999, can be found at www.usdoj.gov/atr/cases/f3800/msjudgex.htm.

competitors. A firm like Netscape has less incentive to develop software applications if Microsoft can develop its own version of these software applications and bundle them with Windows. Perhaps counter-intuitively, the innovation-stifling effect of bundling is worse if Microsoft manages to imitate these applications well. Without strong incentives for software development, some applications may be of inferior quality or may not be developed at all. Given the dominance of Windows and the momentum arising from strong network effects, a threat to the quality or availability of software applications could be quite damaging over the long term. There is thus cause for concern over Microsoft's bundling of applications such as Internet Explorer with Windows.

One remedy, of course, is to prohibit such bundling. Another is to reduce the difference in cost to the consumer of using Internet Explorer rather than Netscape Navigator (c in the model). One way to reduce the cost to the user of using Netscape is to allow original equipment manufacturers (OEMs) to install Netscape on a new PC (one of the allegations against Microsoft is that it has coercively prohibited OEMs from doing this). Even apart from installation, Internet Explorer is more seamlessly integrated into Windows than Netscape Navigator is. This is because Internet Explorer can make use of all application programming interfaces (APIs) available for Windows. APIs allow the developer of application software to invoke blocks of code that are built into the operating system. If Microsoft were compelled to reveal these APIs to software developers, the applications could be integrated as seamlessly as Microsoft's own products. This would reduce or eliminate the cost to the user of using non-Microsoft applications. Since the potential damage to welfare is based on this cost, its reduction or elimination would certainly enhance welfare.

Other goods, such as Adobe Acrobat or Real Player, exhibit some of the same characteristics as Internet browsers. In these cases, understanding the welfare effects of strategies such as bundling requires a consideration of the interrelation of the

components and the network effects between users. This paper is a first step in thinking more generally about such systems of components.

8 Appendix

8.1 Proof of Proposition 1

Given the continuity of the profit functions, there exist prices that satisfy the first order conditions, and $(A1) - (A4)$ ensure that these prices are positive. Thus, the interior equilibrium exists.

Sufficient conditions are given for each equilibrium in which one or both markets tip. For a market to tip, it must be the case that no consumer (or ICP) in that market would be better off by switching. This is only possible if network effects or quality differences are large enough: there must be a positive price that a firm can charge and still keep all consumers or ICPs from switching to the other firm. For a given set of parameters, multiple equilibria may exist. However, taken together, $(A1) - (A4)$ violate all of the following sufficient conditions; if $(A1) - (A4)$ are true, the only equilibrium is the interior equilibrium.

Case 1: $x = 0$, $0 < y < 1$ The equilibrium is $x^* = 0, y^* = \frac{1}{2} - \frac{2m+n+\Delta U_B}{6t}, p^*_{A_2} = \Delta U_A + \frac{m(2m+n+\Delta U_B)}{3t} - t - \delta_b c, p^*_{B_1} = t - \frac{2m+\Delta U_B+n}{3}, p^*_{B_2} = t + \frac{\Delta U_B - 4m+n}{3}$ and the sufficient condition is $\Delta U_A + \frac{m(2m+n+\Delta U_B)}{3t} - t - \delta_b c \geq 0$.

Case 2: $x = 1$, $0 < y < 1$ The equilibrium is $x^* = 1, y^* = \frac{1}{2} + \frac{n - \Delta U_B + 2m}{6t}, p^*_{A_1} = \frac{m(n - \Delta U_B + 2m)}{3t} + \delta_b c - t - \Delta U_A, p^*_{B_1} = t + \frac{n - 4m - \Delta U_B}{3}, p^*_{B_2} = t + \frac{\Delta U_B - 2m - n}{3}$ and the sufficient condition is $\frac{m(n - \Delta U_B + 2m)}{3t} + \delta_b c - t - \Delta U_A \geq 0$.

Case 3: $0 < x < 1$, $y = 0$ The equilibrium is $x^* = \frac{1}{2} + \frac{\delta_b c - \Delta U_A - m - 2n}{6t}$, $y^* = 0$, $p_{A_1}^* = t + \frac{\delta_b c - \Delta U_A - m - 2n}{3}$, $p_{A_2}^* = t + \frac{\Delta U_A + m - \delta_b c - 4n}{3}$, $p_{B_2}^* = \Delta U_B - t + \frac{n(\Delta U_A + m + 2n - \delta_b c)}{3t}$ and the sufficient condition is $\Delta U_B - t + \frac{n(\Delta U_A + m + 2n - \delta_b c)}{3t} \geq 0$.

Case 4: $0 < x < 1$, $y = 1$ The equilibrium is $x^* = \frac{1}{2} + \frac{m + 2n + \delta_b c - \Delta U_A}{6t}$, $y^* = 1$, $p_{A_1}^* = t + \frac{m + \delta_b c - 4n - \Delta U_A}{3}$, $p_{A_2}^* = t + \frac{\Delta U_A - m - \delta_b c - 2n}{3}$, $p_{B_1}^* = \frac{n(m + 2n + \delta_b c - \Delta U_A)}{3t} - t - \Delta U_B$ and the sufficient condition is $\frac{n(m + 2n + \delta_b c - \Delta U_A)}{3t} - t - \Delta U_B \geq 0$.

Case 5: $x = 1$, $y = 1$ The equilibrium is $x^* = 1$, $y^* = 1$, $p_{A_1}^* = m - t + \delta_b c - \Delta U_A$, $p_{B_1}^* = n - t - \Delta U_B$ and the sufficient conditions are $m - t + \delta_b c - \Delta U_A \geq 0$ and $n - t - \Delta U_B \geq 0$.

Case 6: $x = 0$, $y = 0$ The equilibrium is $x^* = 0$, $y^* = 0$, $p_{A_2}^* = m - t - \delta_b c + \Delta U_A$, $p_{B_2}^* = n - t + \Delta U_B$ and the sufficient conditions are $m - t - \delta_b c + \Delta U_A \geq 0$ and $n - t + \Delta U_B \geq 0$.

Case 7: $x = 0$, $y = 1$ The equilibrium is $x^* = 0$, $y^* = 1$, $p_{A_2}^* = m - t - \delta_b c + \Delta U_A$, $p_{B_1}^* = n - t - \Delta U_B$ and the sufficient conditions are $m - t - \delta_b c + \Delta U_A \geq 0$ and $n - t - \Delta U_B \geq 0$.

Case 8: $x = 1$, $y = 0$ The equilibrium is $x^* = 1$, $y^* = 0$, $p_{A_1}^* = m - t + \delta_b c - \Delta U_A$, $p_{B_2}^* = n - t + \Delta U_B$ and the sufficient conditions are $m - t + \delta_b c - \Delta U_A \geq 0$ and $n - t + \Delta U_B \geq 0$.

8.2 Proof of Proposition 2

Let $T_1 = 3t^2 - 2mn - n^2$, $T_2 = 3t^2 - 2mn - m^2$, and $T_3 = 9t^2 - 2n^2 - 5nm - 2m^2$. By $(A1)$, T_1, T_2, and T_3 are positive. Bundling always increases $p_{A_1}^*$ by $\frac{cT_1}{T_2}$ and decreases $p_{A_2}^*$ by $\frac{cT_1}{T_2}$. The changes in $p_{B_1}^*$ and $p_{B_2}^*$ are both equal to $\frac{c(n-m)t}{T_2}$; if $n > m$, $p_{B_1}^*$

increases and $p^*_{B_2}$ decreases, and if $m > n$, $p^*_{B_1}$ decreases and $p^*_{B_2}$ increases. Bundling increases x^* by

$$\frac{ct}{2\left(t^2 - mn\right)} - \frac{2cT_1}{T_2}\left[\frac{t}{2\left(t^2 - mn\right)}\right] - \frac{2c\left(n - m\right)t}{T_2}\left[\frac{m}{2\left(t^2 - mn\right)}\right] \qquad (17)$$

and y^* by

$$\frac{cn}{2\left(t^2 - mn\right)} - \frac{2c\left(n - m\right)}{T_2}\left[\frac{t}{2\left(t^2 - mn\right)}\right] - \frac{2cT_1}{T_2}\left[\frac{n}{2\left(t^2 - mn\right)}\right]. \qquad (18)$$

It is straightforward to verify that these are both positive by $(A1)$, whether $m > n$ or $n > m$. If $n > m$, given the changes in prices and market shares, clearly π_1 increases and π_2 decreases. If $m > n$, $p^*_{B_1}$ falls, but firm 1's market share increases more than it does when $n > m$. It is straightforward to verify that firm 1's profit from B increases with bundling, and clearly firm 1's profit from A increases. Thus bundling increases firm 1's overall profit. A similar argument shows that firm 2's profit decreases with bundling.

8.3   Proof of Proposition 3

Consider the expression for the price of A_1 in the interior equilibrium:

$$\begin{aligned} p^*_{A_1} &= t - n + \frac{\left(\delta_b c - \Delta U_A\right)\left(3t^2 - 2mn - n^2\right) + \Delta U_B\left(n - m\right)t}{9t^2 - 2n^2 - 5mn - 2m^2} & (19) \\ &= t - n + \frac{N}{D}. & (20) \end{aligned}$$

N and D are both of the order n^2, and $\lim\left(\frac{N}{D}\right)_{n\to\infty} = \frac{\delta_b c - \Delta U_A}{2}$. As n increases, the effect of the second term, $-n$, dominates the effect of the third term, $\frac{N}{D}$. For n large enough, the entire expression will be negative. For the same reason, the expression for $p^*_{A_2}$ will be negative for large n. A similar argument applies to $p^*_{B_1}$ and $p^*_{B_2}$.

8.4   Proof of Proposition 4

This proof is very similar to the proof of Proposition 1. Simply set $p_{A_1} = p_{A_2} = 0$ and solve the firms' profit maximization to derive p_{B_1}, p_{B_2}, x^*, and y^*.

8.5   Proof of Proposition 5

Given $\Delta U_A = \Delta U_B = 0$, without bundling, $x^* = y^* = 1/2$. Social welfare is $W_{nb} = U_{A_2} + U_{B_2} + \frac{m+n-t}{2} - c$. By Proposition 1, bundling increases x^* and y^*. Consider two extreme cases: first, $x^* = y^* = 1/2$ with bundling, and second, $x^* = y^* = 1$ with bundling. In both of these cases, bundling welfare is greater. Furthermore, bundling welfare is strictly increasing between these extremes. Therefore, no matter how much bundling increases x^* and y^*, welfare increases.

If $x^* = y^* = 1/2$ under bundling, then $W_b = U_{A_2} + U_{B_2} + \frac{m+n-t-c}{2} > W_{nb}$. If, on the other hand, $x^* = y^* = 1$ under bundling, then $W_b = U_{A_2} + U_{B_2} + m + n - t > W_{nb}$. This follows from the assumed restrictions on c, m, n, and t. Now, to show that W_b is increasing as x^* and y^* increase from $1/2$ to 1 :

Substituting the prices into the expressions for x^* and y^*, we have $x^* = \frac{1}{6} \frac{(3t+2c)(t^2-mn)+t^2c}{t(t^2-mn)}$ and $y^* = \frac{1}{6} \frac{3(t^2-mn)+nc}{t^2-mn}$, or $x^* = y^* + \frac{c(t-n)}{6(t^2-mn)} + \frac{c}{3t}$ and $y^* = x^* - \frac{c(t-n)}{6(t^2-mn)} - \frac{c}{3t}$. Then, substituting for y^* in the expression for W_b and differentiating with respect to x^* yields $\frac{\partial W_b}{\partial x^*} = \frac{nc(m+n)+t^2c+c(t^2-mn)}{3(t^2-mn)}$, which is positive by $(A1)$. Substituting x^* in the expression for W_b and differentiating with respect to y^* yields $\frac{\partial W_b}{\partial y^*} = \frac{2c(m+n)(t^2-mn)+t^2mc}{3t(t^2-mn)}$, which is also positive by $(A1)$.

Since W_b is increasing over this range, for any post-bundling values of x^* and y^* between $1/2$ and 1, W_b is greater than W_{nb}. Bundling welfare is always greater.

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8.6 Proof of Proposition 7

Given $m = n = 0$ and $\Delta U_B = 0$: $p^*_{A_1} = t - \frac{\Delta U_A - \delta_b c}{3}, p^*_{A_2} = t + \frac{\Delta U_A - \delta_b c}{3}, p^*_{B_1} = p^*_{B_2} = t, x^* = \frac{1}{2} - \frac{(\Delta U_A - \delta_b c)}{6t}$, and $y^* = \frac{1}{2}$. The difference in welfare under bundling is

$$W_b - W_{nb} = \frac{c}{2} + \frac{5c^2}{36t} - \frac{7c\Delta U_A}{36t}. \tag{21}$$

Substituting $\Delta U_A > \frac{18t + 5c}{7}$ into (21) yields $W_b - W_{nb} < 0$, which means that welfare is lower with bundling.

8.7 Proof of Proposition 6

The difference in welfare is $\Delta W = W_b - W_{nb}$. When $\Delta U_A = \Delta U_B = 0$,

$$\Delta W = (m + n)\left(2x^* y^* - x^* - y^* + \frac{1}{2}\right) + t\left(-x^{*2} - y^{*2} + x^* + y^* - \frac{1}{2}\right) + cx^*, \tag{22}$$

where $\frac{1}{2} \leq x^*, y^* \leq 1$. Clearly $\frac{\partial \Delta W}{\partial c} > 0$. Given the constraints on x^* and y^*, we have $2x^* y^* - x^* - y^* + \frac{1}{2} > 0$ and $-x^{*2} - y^{*2} + x^* + y^* - \frac{1}{2} < 0$. Therefore, $\frac{\partial \Delta W}{\partial m} > 0$, $\frac{\partial \Delta W}{\partial n} > 0$, and $\frac{\partial \Delta W}{\partial t} < 0$.

8.8 Proof of Proposition 8

From (1)-(4), in the interior equilibrium, bundling changes prices as follows:

$$\Delta p_{A_1} = \frac{c\left(3t^2 - 2mn - n^2\right)}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{23}$$

$$\Delta p_{A_2} = \frac{-c\left(3t^2 - 2mn - n^2\right)}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{24}$$

$$\Delta p_{B_1} = \frac{ct\left(n - m\right)}{9t^2 - 2n^2 - 5mn - 2m^2} \tag{25}$$

$$\Delta p_{B_2} = \frac{-ct\left(n - m\right)}{9t^2 - 2n^2 - 5mn - 2m^2}. \tag{26}$$

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From (5)-(6), market shares change as follows:

$$\Delta x = \frac{(\Delta p_{A_2} - \Delta p_{A_1} + c) t + (\Delta p_{B_2} - \Delta p_{B_1}) m}{2 (t^2 - mn)} \qquad (27)$$

$$\Delta y = \frac{(\Delta p_{B_2} - \Delta p_{B_1} + c) t + (\Delta p_{A_2} - \Delta p_{A_1} + c) n}{2 (t^2 - mn)}. \qquad (28)$$

Thus Δx and Δy are functions of c, m, n, and t only. The difference in welfare due to bundling is

$$-\Delta U_A \Delta x - \Delta U_B \Delta y + (m + n) (2\Delta x \Delta y - \Delta x - \Delta y)$$
$$+ t \left(-\Delta x^2 - \Delta y^2 + \Delta x + \Delta y\right) + c\Delta x, \qquad (29)$$

which clearly is negative if ΔU_A or ΔU_B is large enough.

8.9 Proof of Proposition 9

If firm 2 successfully develops the components, its profit is $p_{A_2} (1 - x^*) + p_{B_2} (1 - y^*)$ and firm 2's expected profit given an investment of R is $p(R) [p_{A_2} (1 - x^*) + p_{B_2} (1 - y^*)] - R$. Assuming firm 2 is risk neutral, it will choose R such that $p'(R) = \frac{1}{p_{A_2}(1-x^*)+p_{B_2}(1-y^*)}$. Let $\pi_{2,nb}$ be firm 2's profit (assuming successful innovation and net of R&D expenditure) when firm 1 does not bundle, and $\pi_{2,b}$ firm 2's profit when firm 1 bundles. If firm 1 is allowed to bundle its inferior version of A, then firm 2's investment, R_b, satisfies $p'(R_b) = \frac{1}{\pi_{2,b}}$, whereas firm 2's investment without bundling, R_{nb}, satisfies $p'(R_{nb}) = \frac{1}{\pi_{2,nb}}$. >From Proposition 2, $\pi_{2,b} < \pi_{2,nb}$, and therefore $p'(R_b) > p'(R_{nb})$. Since $p'' < 0$, $R_b < R_{nb}$.

8.10 Proof of Proposition 10

When there is no bundling, the expected welfare given an investment of R is $p\left(R\right)\left[W_{nb}\right]-R$, and a social planner seeking to maximize welfare will choose R_{nb}^{*} such that $p'\left(R_{nb}^{*}\right)=\frac{1}{W_{nb}}$.Now, social welfare has to be greater than firm 2's (post-investment) profit: the profit itself is a transfer, and at least some consumers and ICPs gain surplus from participating in the market. Therefore, without bundling, firm 2 will invest less in R&D than is socially optimal (again, since $p''<0$, $R_{nb}<R_{nb}^{*}$). If there is bundling, the social planner chooses R_{b}^{*} such that $p'\left(R_{b}^{*}\right)=\frac{1}{W_{b}}$. Again, firm 2's profit is less than social welfare, and so $R_{b}<R_{b}^{*}$. We always have $R_{nb}>R_{b}$ and $R_{nb}^{*}>R_{nb}$. If $W_{b}>W_{nb}$, then $R_{b}^{*}>R_{nb}^{*}$. Putting all this together, $R_{b}^{*}>R_{nb}^{*}>R_{nb}>R_{b}$. This implies $R_{b}^{*}-R_{b}>R_{nb}^{*}-R_{nb}$, which means that firm 2's R&D incentive is further from the social optimum under bundling. Bundling exacerbates the inefficiency in firm 2's investment decision.

If $W_{nb}>W_{b}$, then $R_{nb}^{*}>R_{b}^{*}$. In this case, whether or not there is bundling, firm 2 has too little incentive to invest in R&D: $R_{b}^{*}>R_{b}$ and $R_{nb}^{*}>R_{nb}$. But we cannot determine whether $R_{b}^{*}-R_{b}$ is greater or less than $R_{nb}^{*}-R_{nb}$; it is not clear whether bundling increases or decreases the difference in the privately and socially optimal levels of investment.

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